Dynamics of Utility Based Hedging Strategy

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Abstract

We reviewed the utility based option trading and hedging approach as well as other results under the asymptotic analytical approximation method and introduced the option hedging problem which clearly illustrates the intuition behind the hedging bandwidth and volatility adjustment. However, we used the multi-period measure determine the absolute risk aversion to formulate a dynamic spectrum of variation for the market risk. Hence, determine the best hedging strategy under the frame work of utility based hedging method, the hedgers value function, market volatility, the rate of purchase (call) and sales (put) on risky assets with sufficient precision.

Key Words: *Utility Based, Hedging Strategy, Multi-period Measure, Absolute Risk Averse and Market Volatility*

INTRODUCTION

A variety of approaches have been suggested to deal with the problem of option pricing and hedging with transaction costs. (**See Clewlow and Hodges (1997), Martellini and Priaulet (2002), and VeleriZakamouline (2004)**). However, their numerical algorithm is cumbersome to implement and the calculation of the optimal hedging strategy is time consuming. However, in modern finance it is customary to describe risk preferences by a utility function. The expected utility theory maintains that individuals behave as if they were maximizing the expectation of some utility function in all possible outcomes. **Hodges and Neuberger (1989)** pioneered the option pricing and hedging approach based on this theory. According to the utility-based approach, the qualitative description of the optimal hedging strategy is as follows: do nothing when the hedge ratio lies within a so-called "no transaction region" and rehedge to the nearest boundary of the no transaction region as soon as the hedge ratio moves out of the no transaction region. One commonly used simplification of the optimal hedging strategy, widely used in practice, is known as hedging to a fixed bandwidth around delta $\Delta = \frac{\partial V}{\partial S} \pm H$. This strategy prescribes to rehedge when the hedge ratio moves outside of the prescribed tolerance from the corresponding Black-Scholes delta. Since there are no explicit solutions for the utility-based hedging with transaction costs and the numerical methods are computationally hard. For practical applications, it is of major importance to use other alternatives. One of such alternatives is to calibrate a rehegding function when some parameters in the problem assume large or small values. **Whalley and Wilmott (1997)** were the first to provide this analysis of the model **of Hodges and Neuberger (1989)** assuming that transaction costs are small**. Barles and Soner (1998)** performed an alternative asymptotic analysis of the same model assuming that both the transaction costs and the hedger's risk tolerance are small. However, the results of **Barles and Soner (1998)** are quite different from those of **Whalley and Wilmott (1997). Whalley and Wilmott (1997**) derive only an optimal form of the hedging bandwidth centered around the Black-Scholes +delta, but with different delta (adjusted price of the option) specification.

The Utility-Based Hedging Strategy

Here, we reviewed the utility based option trading and hedging approach as well as other results under the asymptotic analytical approximation method and introduced the option hedging problem which clearly illustrates the intuition behind the hedging bandwidth and volatility adjustment. However, the starting point for the utility-based option pricing and hedging approach is to consider the optimal portfolio selection problem of the hedger who faces transaction costs and maximizes the expected utility of his terminal wealth. The hedger has a finite time horizon$[t;\ T]$, x_t in the bank account, and y_t shares of the stock at time *t*. S_t and S_T are the underlying asset prices at $[t;\ T]$. The value function of the hedger with and with no option liability is defined as

$$Jw(t, x_t, y_t, S_t, K) = maxE_t[U(x_T + y_T ST - (ST - K)^+)]\,. \qquad (1)$$

And

$$J_0(t, x_t, y_t, S_t) = maxE_t[U(x_T + y_T S_T)], \qquad (2)$$

Where

$$U_\alpha = U(x_T + y_T S_T). \qquad (3)$$

Is the utility value function.

THE HEDGING PROBLEM

Consider a continuous time economy with one risk-free and one risky asset, which pays no dividends. We will refer to the risky asset as the stock, and assume that the price of the stock, *St*, evolves according to a diffusion process given by

$$dSt = \mu Stdt + \sigma StdWt; \qquad (4)$$
$$dSt/St = \mu dt + \sigma dWt$$

where μ and σ are, respectively, the mean and volatility of the stock returns per unit of time, and *Wt* is a standard Brownian motion. The risk-free asset, commonly referred to as the bond or bank account, pays a constant interest rate of $r \geq 0$. We consider hedging a short option with maturity T and strike price K. We assume that a purchase or sale of δS shares of the stock incurs transaction costs $\lambda|\delta S|$ proportional to the transaction ($\lambda \geq 0$). Denote the value of the option at time *t* as

$$V(S_t, t) = Ke^{-r(T-t)}. \qquad (5)$$

Where $\delta(t, T)$ is the discount factor given by

$$\delta(t,T) = e^{-r(T-t)} \tag{6}$$

The terminal payoff of the option one wishes to hedge is given by

$$V\,(S_T,T\,) \;=\; max\,\{ST - K, 0\} \;=\; (S_T \;-\; K)^+\,, \tag{7}$$

As the stock price attains maximum,

$$V(S_T,T) = Ke^{-\left(r+\frac{3}{2}\sigma^2\right)(T-t)}. \tag{8}$$

According to **Avellaneda et al 1994**, when a hedger writes an option, he receives the value of the option $V\,(S_t,t)$ and sets up a hedging portfolio by buying Δ shares of the stock and putting V $(S_t,t)-\Delta(1+\lambda)S_t$ in the bank account. As time goes, the writer rebalances the hedging portfolio according to some prescribed rule;

i) With the unconditional Sharpe ratio of the hedged portfolio at maturity i.e. $V(S_T,T)$.

ii) with the certainty equivalent growth rate of the terminal wealth as measured by utility $U(\alpha)$ is

$$U(\alpha) = -e^{-\gamma\alpha};\quad \gamma > 0. \tag{9a}$$

$U(\alpha)$ is the hedger's utility function and it is assumed that the hedger has a negative or positive utility function, where γ is a measure of the hedger's absolute risk aversion. **(VeleriZaka, 2004).** This particular choice of utility function might seem restrictive. However, as it was conjectured by **Davis et al. (1993)** and showed in **Andersen and Damgaard (1999),** an option price is approximately invariant to the specific form of the hedger's utility function, and mainly, only the level of absolute risk aversion plays an important role**. (Zaka, 2004**).

In the Black-Scholes model, the risk position at time S_t is modeled by a geometric Brownian motion that is $S_0e^{(\mu-\frac{\sigma^2}{2})}\ t_0\ W_t\ t \geq o.$

Proposition 1 Let Z be a standard normal variable and X be a transformation of

Z: X = h(z), Then $H(X;\alpha) = E[h(z+\alpha)]$,which implies that

$$H(X;\alpha) = \int_0^\infty P[h(Z+\alpha) > t]\,dt \;= E[h(Z+\alpha)] = S_0e^{z_T}{+}\alpha \tag{9b}$$

Where h is a continuous positive and decreasing function. It is straight to show that for a normal random variable Z.

$$H[X = h(Z){:}\,\alpha] = \; E[h(Z+\alpha)]$$

$$E[X = h(Z){:}-\alpha] => H[S_T - \alpha] = S_0\mathrm{e}^{\mathrm{Z_T}+\alpha_\mathrm{T}} \tag{9c}$$

If S_t is the price of a security at time, t following a geometric Brownian motion so that

$$S_t = S_0e^{\left(\mu-\frac{\sigma^2}{2}\right)t+\sigma W_t}$$

Where W_t is a Brownian motion under P then S_T can be written as a function of the standard normal random variable Z. In this case $S_T = h(Z)$

$$h(x) = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}x}$$

Applying the kernel we have $H(S_T, \alpha) = E[h(Z - \alpha)]$

$$E(S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}z - \sigma\sqrt{T}} = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\alpha + \frac{\sigma^2}{2}}$$

For $\alpha = \frac{\mu - r_c}{2}$ simplifies to $H(S_T, \alpha) = S_0 e^{-r_c} T$

then the current price becomes $e^{-r_c}\mathrm{T}\ H(S_T, \alpha) = e^{-r_c} T S_0 e^{-r_c} T = S_0$. Thus, the parameter α calibrates the discounted certainty equivalent of the security price on future date to the initial price of security. If we consider the pay-off of an European call option (with maturity T and strike price k) we have

$$S_T = C(S_T, k) = (S_T - k) \tag{9d}$$

Where S_T is a lognormal random variable. Applying the kernel to this payoff with

$$\alpha = \frac{\pi - rc\sqrt{T}}{\sigma} \qquad \text{then,}$$

$$e^{-rT} H[C(X_T, K): -\alpha] = S_0 \phi(\ln\left(\frac{x_0}{K}\right) + \left(\frac{r+\theta^2}{2}\right)T - e^{-rT} k\phi \frac{\ln\left(\frac{x_0}{K}\right) + \left(\frac{r+\theta^2}{2}\right)T}{\sigma\sqrt{T}} - \sqrt[\sigma]{T}$$

$$= S_0 \phi\left(\ln\left(\frac{x_0}{K}\right) + \left(\frac{r+\theta^2}{2}\right)\right)T - e^{-rT} k\phi \frac{\ln\left(\frac{x_0}{K}\right) + \left(\frac{r+\theta^2}{2}\right)}{\sigma\sqrt{T}} - \sqrt[\sigma]{T} \tag{9e}$$

And r is the risk free rate, which shows the Black-Scholes model for option trading.

Where

$$\frac{\ln\left(\frac{x_0}{K}\right) + \left(\frac{r+\theta^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{\partial V}{\partial S} \tag{9f}$$

Hedging To A Fixed Bandwidth

One commonly used simplification of the optimal hedging strategy is known as hedging to a fixed bandwidth around delta. This strategy prescribes to rehedge when the hedge ratio moves outside of the prescribed tolerance from the corresponding Black-Scholes delta. More formally, the boundaries of the no transaction region are defined by

$$\Delta = \frac{\partial V}{\partial S} \pm H \tag{10}$$

Where $\frac{\partial V}{\partial S}$ is the Black-Scholes hedge, and H is a given constant tolerance. The intuition behind this strategy is obvious: the parameter H is a proxy for the measure of risk of hedging portfolio.

More risk avers hedger would choose a low H, while more risk tolerance hedgers will accept a higher value of H.
In the frame work of the utility based hedging approach, the option hedging strategy Is defined as the difference, $y_w(\tau) - y_0(\tau)$, between the hedger's optimal trading strategies with and without option liability. In the absence of transaction costs, the optimal number of shares the hedger would hold without and with option liability are given by **(Davis et al. 1993).**

$$y_0 = \frac{\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2}, \tag{11}$$

$$y_w = \frac{\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2} + \frac{\partial V}{\partial S}, \tag{12}$$

Consequently, the option hedging strategy in the absence of transaction cost is simply the blackscholes strategy where *the absolute risk parameter* γ *is largely unknown*;

$$\Delta = y_w - y_0 = \frac{\partial V}{\partial S}. \tag{13}$$

$$\frac{\partial V}{\partial S} = N(d_1), \tag{14}$$

Where

$$N(d_1) = \frac{log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \quad . \tag{15}$$

Whalley and Wilmott (1997) show the boundaries of the no transaction region as;

$$\Delta = \frac{\partial V}{\partial S} \pm H' = N(d_1) \pm \left(\frac{3}{2}\frac{e^{-r(T-t)}\lambda S T^2}{\gamma}\right)^{\frac{1}{2}}. \tag{16}$$

Barles and Soner (1998) performed an alternative asymptotic analysis of same model assuming that both the transaction costs and hedgers risk tolerance are small. They find that the optimal hedging strategy is to keep the hedge ratio inside the no transaction region is given by

$$\Delta = \frac{\partial V}{\partial S} \pm H'' = \frac{\partial V((\sigma_m)}{\partial S} \pm \frac{1}{\lambda\gamma S} g(\lambda^2\gamma S^2 \Gamma) \tag{17}$$

Where $\frac{\partial V((\sigma_m)}{\partial S}$ is the black scholes hedge with an adjusted volatility

$$\sigma_m^2 = \sigma^2\left(1 + f\left(e^{r(T-t)}\lambda^2\gamma S^2 \Gamma\right)\right). \tag{18}$$

The sensitivity of the option delta to the underlying asset price, is known as gamma.

$$\Gamma = \frac{N(d_1)}{S\sigma\sqrt{(T-t)}}. \tag{19}$$

To decrease the amount of transaction costs, it makes sense to decrease the option gamma, at least in regions where it is high, thus making the option delta a flatter function of the underlying asset.

The optimal trading policies in the presence and absent of transaction costs suggest the following general specifications of the hedgers no transaction region without and with option liability;

$$y_0(\tau) = \frac{\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2} \pm H_o \tag{20}$$

$$y_w(\tau) = \frac{\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2} + \frac{\partial V}{\partial S} \pm (H_o + H_w)\,, \tag{21}$$

In the presence of transaction cost;

$$y_0(\tau) = \frac{K\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2} \pm H_o, \tag{22}$$

$$y_w(\tau) = \frac{K\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2} + \frac{\partial V}{\partial S} \pm (H_o + H_w)\,, \tag{23}$$

H_o is half of the width of the no transaction region without option liability, H_w is an additional increase in the width of the no transaction region induced by the presence of an option. The two boundaries $H_o\ and\ H_w$ are computed in accordance with: for a fixed set parameters $T - t, r, \sigma, \lambda, \gamma S, \frac{S}{K}$, numerically. Then

$$H_o = \frac{y_u^0 - y_l^o}{2} \quad , \tag{24}$$

$$H_w = \frac{y_u^w - y_l^w}{2} - H_o \quad . \tag{25}$$

Here, $2H_o = y_u^o - y_l^o$, And $2H_w + H_0 = y_u^w - y_i^w$.

$y_u's\ and\ y_l's$ are the upper and lower boundaries of the hedging bandwidth without and with option liability, given.

H_w Obviously depends on the option gamma. In other words, when the option gamma approaches zero, the width of the no transaction region with option liability becomes equal to that of without option liability.

The functional form of the approximating function for $H_o\ and\ H_w$ are given to be;

$$H_o = \alpha\sigma^{\beta_1}\lambda^{\beta_2}(\gamma S)^{\beta_3}\delta(t.T)^{\beta_4}(T-t)^{\beta_5}, \tag{26}$$

$$H_w = \alpha r^{\beta_1}\sigma^{\beta_2}\lambda^{\beta_3}(\gamma S)^{\beta_4}(N(d_1))^{\beta_5}\delta(t.T)^{\beta_6}(T-t)^{\beta_7}(e^{(T-t)})^{\beta_8}. \tag{27}$$

$N(d_1)$ is the cardinality of the Black- Scholes hedge, with the volatility σ, and $\beta_i's$ are the boundary space.

Whalley and Willmolt (1997) as well **as Barles and Soner (1998)** used the numerical and asymptotic analytical approximation method (AAM) to reveal the underlying structure of the solution under realistic fixed model parameter but there is no explicit measure value to the absolute risk aversion parameter. However, their numerical computational algorithm is cumbersome to implement and time consuming. The option hedging strategy in the absence of transaction cost is simply the Black-Scholes. Consequently, the risk aversion parameter is largely unknown. Hence, failure of the sharpe ratio to capture the true nature of investment opportunities. To this, one cannot carry on the measurement of the hedging bandwidth, hedgers' value function, volatility size and rate of transaction with sufficient precision.

MODEL FORMULATION

We Consider the average multiperiod dimensional measure as the optimal extraction part to be

$$D_{MH} \quad \triangleq \bar{f} = \frac{1}{\Delta\alpha}\int_0^\infty F(\alpha)d\alpha \ .$$

Let $(R^n, \beta(R^n))$ be a measurable space and $\bar{f}: \beta(R^n) \to R$ be a measurable function. $\bar{f} \subset \beta(R^n)$ With the gauge function

$$\triangleq \bar{f}_\alpha(a,b) = \frac{1}{\Delta\alpha}\int_0^\infty F(\alpha)d\alpha \ . \qquad (28)$$

Here, $\bar{f}$ is assumed to be the multifractal expectation at any given confidence level α.
$\Delta\alpha$ Is the difference in singularity strength of market indices and $F(\alpha)$ is the multifractal function.
Here,

$$\Delta\alpha = 1 - \alpha \qquad (29)$$

$$F(\alpha) = V(S_T, T)(U(\alpha)). \qquad (30)$$

We establish the gauge function to be;

$$\bar{f}_\alpha(0,1) \ = \left| \frac{1}{\gamma\Delta\alpha} \right| ; \gamma > 0 \qquad (31)$$

So that

$$\gamma = \left| \frac{1}{\bar{f}_{\alpha(0,1)}\Delta\alpha} \right|. \qquad (32)$$

Lemma

The absolute risk aversion parameter for the continuous HARA utility based option hedging and trading strategy, in the presence of the underlying market prices, is given by;

$$\gamma = \left| \frac{V(S_T,T)}{\bar{f}_\alpha(0,1)\Delta\alpha} \right|. \qquad (33)$$

Similarly, in the absence of the underlying market prices we have,

$$\gamma = \left| \frac{1}{\bar{f}_{\alpha(0,1)}\Delta\alpha} \right|. \qquad (34)$$

We establish that the D-dimensional multiperiod measure under HARA on

a set $\bar{f}_{\alpha(0,1)} \triangleq \{a: 0 < D < 1\}$ is given by

$$M_D(\bar{f}_\alpha) = \lim_{\varepsilon\to 0}\sum_k r_k^D ; D > 0 ; r_k < \varepsilon. \qquad (35)$$

Define the optimal covering of this set using variable radius, r_k. The multiperiod HARA – dimension measure D_{MH} is the value of $D(\gamma)$ at which $M_D(\alpha)$ changes within the set of interval $(0,1)$, while the dimension D can be calculated as (1.22) i.e.

$$D = \frac{\log N_1}{\log S_1} \qquad (36)$$

(whereN_1 is the number of small pieces that go into the larger one and S_1 is the scale to which the smaller pieces compare to the larger one). Equivalently for a given precision level, $\varepsilon > 0$, $N_1(\varepsilon)$ satisfies a power law as $\varepsilon \to 0$ so that

$$N_1(\varepsilon) \sim \varepsilon^{-D}.$$

D is a constant called the fractal dimension, which helps to analyze the structure of a fixed multi-fractal.For a large class multi-fractals, the dimension $D(\alpha)$ coincides with the multi-fractal spectrum. For any $\alpha \geq 0$, the set $F(\alpha)$ can be defined as the HARA exponent α with a fractal dimension $D(\alpha)$ satisfying $0 < D(\alpha) < 1$.

Let $(R^n, \beta(R^n))$ be a measurable space and$\bar{f}: \beta(R^n) \to R$ be a measurable function.

Let α be a real valued function on $\beta(R^n)$, then the multiperiod spectrum with respect to the functions $\gamma \ and \ \alpha$ is given by;

$$D(\alpha) = \inf\big(\varepsilon \, \epsilon R : \bar{f}(\alpha)\big) \leq \Delta\alpha. \tag{37}$$

ESTIMATION OF PARAMETERS

Similarly, from (9), we establish that;

$$U(\alpha) = -e^{-[\frac{1}{\bar{f}(\Delta\alpha)}]\alpha}; \quad \gamma > 0.$$

(16) becomes;

$$\Delta = N(d_1) \pm (\frac{3}{2} e^{-r(T-t)} \lambda S T^2 \bar{f} \Delta\alpha)^{\frac{1}{2}}$$

Where (17) becomes;

$$\Delta = \frac{\partial V(\sigma_m)}{\partial S} \pm \frac{\overline{f(\Delta\alpha)}}{\lambda S} g(\lambda^2 [\frac{S^2 \Gamma}{\overline{f(\Delta\alpha)}}]).$$

Calibrating (33) into (17), $\sigma_m \ and \ \sigma$ can be estimated as:

$$\sigma = \sqrt{\frac{\sigma_m^2}{\left(1 + f(\delta(t,T)\lambda^2 S^2 \Gamma / \bar{f}\Delta\alpha)\right)}} \ . \tag{38}$$

From σ, the rate of purchase (call) and sales (put) becomes,

$$\lambda = \sqrt{\frac{\sigma^2 - \bar{f}\Delta\alpha(\sigma_m^2)}{\sigma^2 f(\delta(t,T) S^2 \Gamma}} \ . \tag{3.14}$$

Then,

$$S = \sqrt{\frac{\sigma^2 - \bar{f}\Delta\alpha(\sigma_m^2)}{\sigma^2 f(\delta(t,T)\lambda^2 \Gamma}} \quad . \tag{39}$$

So that the functional form of H_o *and* H_w becomes;

$$H_o = \alpha\left(\sqrt{\frac{\sigma_m^2}{\left(1+f(\,\delta(t,T)\lambda^2\, S^2\Gamma/\bar{f}\Delta\alpha\,)\right)}}\right)^{\beta_1} \lambda^{\beta_2}(\gamma S)^{\beta_3}\delta(t.T)^{\beta_4}(T-t)^{\beta_5}, \tag{40}$$

$$H_w = \alpha r^{\beta_1}\left(\sqrt{\frac{\sigma_m^2}{\left(1+f(\,\delta(t,T)\lambda^2\, S^2\Gamma/\bar{f}\Delta\alpha\,)\right)}}\right)^{\beta_2} \lambda^{\beta_3}(\gamma S)^{\beta_4}(N'(d_1))^{\beta_5}\delta(t.T)^{\beta_6}(T-t)^{\beta_7}(e^{(T-t)})^{\beta_8}.$$

(41)

Hedging to a fixed bandwidth around delta

Recall that the option hedging strategy is defined as the difference between the hedger's optimal trading strategies with and without option liability $ie\ y_w(\tau) - y_0(\tau)$. In the absence of transaction costs, calibrating (33) into (11) and (12), the solutions for the optimal number of shares the hedger would hold without and with option liability becomes;

$$y_0 \;=\; \frac{\delta(t,T)(\mu-r)\bar{f}(\Delta\alpha)}{S\sigma^2} + \frac{\partial V}{\partial S}, \tag{42}$$

To this, $$y_w - y_0 = \frac{\partial V}{\partial S} \quad a.s.$$

Under the optimal trading policy, for general specification of the hedgers no transaction region we have;

$$y_w(\tau) - y_0(\tau) = = \frac{\partial V}{\partial S} \pm (H_o + H_w) \pm H_o. \tag{43}$$

As H_w approaches zero,

$$y_w(\tau) - y_0(\tau) \;= \frac{\partial V}{\partial S} \pm (H_o) \pm H_o \;,$$

$$= \frac{log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \pm (y_u^0 - y_l^o) \;.$$

Calibrating (39) and (40) into (43), we have;

$$y_w(\tau) - y_0(\tau) \;\; \frac{log\left(\frac{S}{K}\right)+\left(r+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \pm \alpha\left(\sqrt{\frac{\sigma_m^2}{\left(1+f\left(\frac{\delta(t,T)\lambda^2\, S^2\Gamma}{\bar{f}\Delta\alpha}\right)\right)}}\right)^{\beta_1} \lambda^{\beta_2}(\gamma S)^{\beta_3}\delta(t.T)^{\beta_4}(T-t)^{\beta_5} + \alpha r^{\beta_1}\left(\sqrt{\frac{\sigma_m^2}{\left(1+f\left(\frac{\delta(t,T)\lambda^2\, S^2\Gamma}{\bar{f}\Delta\alpha}\right)\right)}}\right)^{\beta_2} \lambda^{\beta_3} \pm \left({}^{1}\!/_{\bar{f}}\left[\frac{1}{\Delta\alpha}\right]\right) S)^{\beta_4}(N'(d_1))^{\beta_5}\delta(t.T)^{\beta_6}(T-t)^{\beta_7}(e^{(T-t)})^{\beta_8} \pm \alpha\left(\sqrt{\frac{\sigma_m^2}{\left(1+f\left(\frac{\delta(t,T)\lambda^2\, S^2\Gamma}{\bar{f}\Delta\alpha}\right)\right)}}\right)^{\beta_1} \lambda^{\beta_2}(\gamma S)^{\beta_3}\delta(t.T)^{\beta_4}(T-t)^{\beta_5}. \qquad (44)$$

Then (3.21) becomes;

$$\frac{log\left(\frac{S}{K}\right)+\left(r+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}} \pm 2\alpha\left(\sqrt{\frac{\sigma_m^2}{\left(1+f\left(\frac{\delta(t,T)\lambda^2\, S^2\Gamma}{\bar{f}\Delta\alpha}\right)\right)}}\right)^{\beta_1}\left(\sqrt{\frac{\sigma^2+\bar{f}\Delta\alpha(\sigma_m^2)}{\sigma^2 f(\delta(t,T)S^2\Gamma}}\right)^{\beta_2} \pm \left({}^{1}\!/_{\bar{f}}\left[\frac{1}{\Delta\alpha}\right]\right)\left(\sqrt{\frac{\sigma^2+\bar{f}\Delta\alpha(\sigma_m^2)}{\sigma^2 f(\delta(t,T)\lambda^2\Gamma}}\right))^{\beta_3}\delta(t.T)^{\beta_4}(T-t)^{\beta_5}\,. \qquad (45)$$

CONCLUSION

This approach provides a simpler technique and policy for predicting the optimal portfolio investment policies. Meaning that, more risk averse hedger would choose a low constant tolerance (3.20), while more risk tolerance hedger will accept a higher value of it (3.21). To this, the management of the monetary amount invested in the risky asset through time, is independent of the total wealth but depends on the absolute risk aversion.

44- : To the risk averse hedger; the width and increase in width of the option lies within the boundaries of the no transaction region. This means that, increase in γ, increases the hedging bandwidth and decreases the risk of the hedged portfolio.

45-: To the risk tolerance hedger; twice the width of the option lies within the boundaries of sthe no transaction region. Meaning that, decrease in γ, decreases the hedging bandwidth and increases the risk of the hedged portfolio.

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